Total Variation Regularized Frechet Regression for Metric-Space Valued Data

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Abstract: Non-Euclidean data that are indexed with a scalar predictor such as time are increasingly encountered in data applications, while statistical methodology and theory for such random objects are not well developed yet. To address the need for new methodology in this area, we develop a total variation regularization technique for nonparametric Frechet regression, which refers to a regression setting where a response residing in a generic metric space is paired with a scalar predictor and the target is a conditional Frechet mean. Specifically, we seek to approximate an unknown metric-space valued function by an estimator that minimizes the Frechet version of least squares and at the same time has small total variation, appropriately defined for metric-space valued objects. We show that the resulting estimator is representable by a piece-wise constant function and establish the minimax convergence rate of the proposed estimator for metric data objects that reside in Hadamard spaces. We illustrate the numerical performance of the proposed method for both simulated and real data, including the metric spaces of symmetric positive-definite matrices, probability distributions and phylogenetic trees.