SLOPE meets AMP: Does SLOPE outperform LASSO?

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Abstract: In high-dimensional problem of reconstructing a sparse signal via the sorted L1 penalized estimation, or SLOPE, we apply the approximate message passing (AMP) to SLOPE problem.

We derive the AMP algorithm and state evolution respectively. We then rigorously prove that AMP solution converges to SLOPE minimization solution as iteration increases. We also use the state evolution for non-separable functions to asymptotically characterize the SLOPE solution.

As a consequence, AMP and state evolution allow us to conduct inference on the SLOPE solution and demonstrate cases where SLOPE is better than LASSO (which is a special case of SLOPE). Our first result is the trade-off between false and true positive rates or, equivalently, between measures of type I and type II errors along the SLOPE path. Especially, LASSO is known to suffer from Donoho-Tanner phase transition where TPP may be bounded away from 1. In contrast, SLOPE overcomes such phase transition and one of the path can be nicely characterized as a Mobius transformation. Our second result considers fixed signal prior distribution and constructs SLOPE path that has better TPP, FDP and MSE at the same time.