Regularization of High-Dimensional Toeplitz Covariance Structure via Entropy Loss Function

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Abstract: The estimation of structured covariance matrix arises in many applications. An appropriate covariance structure not only improve the accuracy of covariance estimation but also increase the efficiency of the mean parameter estimation in statistical models. For example, a good estimation of covariance structures leads to accurate trajectory predictions for longitudinal data and time series data. In this paper we propose a novel statistical method that is able to select the optimal Toeplitz structure and estimate the high-dimensional covariance matrix simultaneously. Entropy loss functions with nonconvex penalties are employed as matrix-discrepancy measures, under which the optimal covariance structure and the selection of the associated Toeplitz structures are made, simultaneously. The resulting Toeplitz structured covariance estimators are guaranteed to be positive definite, unbiased and selection consistent. Asymptotic theories are derived and simulation studies are conducted, showing a very high accurate Toeplitz covariance structure estimation. The proposed method is also applied to three real data practices, demonstrating its good performance in covariance estimation in practice.