A geometric perspective of hypothesis testing

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Abstract: This talk is devoted to understanding the behavior of compound testing problems within the Gaussian sequence model from a geometric perspective. In this talk, two vignettes are considered.

When the null and alternative are specified by a pair of closed, convex cones---the case arising in various applications, including detection of treatment effects, trend detection in econometrics, signal detection in radar processing, and shape-constrained inference in non-parametric statistics, we studied the behavior of the generalized likelihood ratio test (GLRT). Despite the wide-spread use of the GLRT, its properties have yet to be fully understood. When is it optimal, and when can it be improved upon? How does its performance depend on the cones? I provide some answers to these and other questions, all based on a tight characterization of the GLRT's performance.

When the hypotheses are given by a known vector within a high dimensional ellipse, or other unknown vectors in the ellipse---the case underlying the heart of non-parametric goodness-of-fit testing, signal detection in cognitive radio, and regression function testing in reproducing kernel Hilbert spaces, we study difficulty in a way that is adaptive to vectors within the ellipse. By characterizing the localized minimax testing radius sharply, our results yield interesting phenomena that were not known before.