Bayesian variance estimation in the Gaussian sequence model with partial information on the means

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Abstract: Consider the Gaussian sequence model under the additional assumption that a fraction of the means are known. We study the problem of variance estimation from a frequentist Bayesian perspective. The maximum likelihood estimator (MLE) for the variance is biased and inconsistent. This raises the question whether the posterior is able to correct the MLE in this case. By developing a new proving strategy that uses refined properties of the posterior distribution, we find that the marginal posterior is inconsistent for any i.i.d. prior on the mean parameters. In particular, no assumption on the decay of the prior needs to be imposed. Surprisingly, we also find that consistency can be retained for a hierarchical prior based on Gaussian mixtures. In this case we also establish a limiting shape result and determine the limit distribution. In contrast to the classical Bernstein-von Mises theorem, the limit is non-Gaussian. By conducting a small numerical study, we show that the Bayesian analysis leads then to new statistical estimators outperforming the correctly calibrated MLE in a numerical simulation study.