Fixed support positive-definite modification of covariance matrix estimators via linear shrinkage

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Abstract: We study the positive definiteness (PDness) problem in covariance matrix estimation. For high-dimensional data, many regularized estimators have been proposed under structural assumptions on the true covariance matrix, including sparsity. They were shown to be asymptotically consistent and rate-optimal in estimating the true covariance matrix and its structure. However, many of them do not take into account the PDness of the estimator and produce a non-PD estimate. To achieve PDness, researchers considered additional regularizations (or constraints) on eigenvalues, which make both the asymptotic analysis and computation much harder. In this paper, we propose a simple modification of the regularized covariance matrix estimator to make it PD while preserving the support. We revisit the idea of linear shrinkage and propose to take a convex combination between the first-stage estimator (the regularized covariance matrix without PDness) and a given form of diagonal matrix. The proposed modification, which we call the FSPD (Fixed Support and Positive Definiteness) estimator, is shown to preserve the asymptotic properties of the first-stage estimator if the shrinkage parameters are carefully selected. It has a closed form expression and its computation is optimization-free, unlike existing PD sparse estimators. In addition, the FSPD is generic in the sense that it can be applied to any non-PD matrix, including the precision matrix. The FSPD estimator is numerically compared with other sparse PD estimators to understand its finite-sample properties as well as its computational gain. It is also applied to two multivariate procedures relying on the covariance matrix estimator - the linear minimax classification problem and the Markowitz portfolio optimization problem - and is shown to improve substantially the performance of both procedures.