The randomization distribution of the logrank statistic

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Abstract: The logrank test is one of the most popular approaches for comparing time-to-event outcomes in the presence of censoring. Most theoretical justifications given for it have required a hypothetical superpopulation, in the sense that the event times for all units are independent and identically distributed (i.i.d.). We invoke the potential outcome framework to define the causal effect of certain treatment on a time-to-event outcome, and conduct finite population inference that relies crucially on the physical randomization of the treatment. On the one hand, finite population inference focuses particularly on the finite experimental units by viewing their potential outcomes as fixed constants; this is equivalent to conducting inference conditioning on all the potential outcomes. On the other hand, the test justified by finite population inference can be valid without any distributional assumptions (such as i.i.d.) on the potential outcomes. In this paper, we study finite population inference for the logrank test, and specifically we investigate the randomization distribution of the logrank statistic. We show that, under a Bernoulli randomized experiment with non-informative i.i.d. censoring within each treatment arm, the logrank test is asymptotically valid for testing Fisher's null hypothesis of no treatment effect on any unit. The asymptotic validity of the logrank test does not require any distributional assumptions on the potential event times; for example, the potential event times can have arbitrary dependence and heterogeneity across the units. The developed theory for the logrank rank test from finite population inference supplements its classical theory from usual superpopulation inference, and thus helps provide a broader justification for the logrank test. We also extend the theory to the stratified logrank test, which is useful for randomized blocked designs and when censoring mechanisms vary across strata.