Limit distribution theory in multiple isotonic regression

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Abstract: We study limit distributions for the tuning-free max-min block estimators in multiple isotonic regression under both fixed lattice design and random design settings. We show that at a fixed interior point in the design space, the estimation error of the max-min block estimator converges in distribution to a non-Gaussian limit at certain rate depending on the number of vanishing derivatives and certain effective dimension and sample size that drive the asymptotic theory. The limiting distribution can be viewed as a generalizing the well-known Chernoff distribution in univariate problems. The convergence rate is optimal in a local asymptotic minimax sense. There are two interesting features in our local theory. First, the max-min block estimator automatically adapts to the full spectrum of local smoothness levels and the intrinsic dimension of the isotonic regression function at the optimal rate. Second, the optimally adaptive local rates are in general not the same in fixed lattice and random designs. In fact, the local rate in the fixed lattice design case is no slower than that in the random design case, and can be much faster when the local smoothness levels of the isotonic regression function or the sizes of the lattice differ substantially along different dimensions. This is joint work with Qiyang Han.