Gaussian unitary ensembles with pole singularities near the soft edge and a system of coupled Painlevé XXXIV equations

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Abstract: In this talk, we consider the singularly perturbed Gaussian unitary ensembles defined by the measure begin{equation*} frac{1}{C_n} e^{- ntextrm{tr}, V(M;lambda,vec{t};)}dM, end{equation*} over the space of $n times n$ Hermitian matrices M, where $V(x;lambda,vec{t};):= 2x^2 + sum_{k=1}^{2m}t_k(x-lambda)^{-k}$ with $vec{t}= (t_1, t_2, ldots, t_{2m})in mathbb{R}^{2m-1} times (0,infty)$, in the multiple scaling limit where $lambdato 1$ %approaches the soft edge of the limiting spectrum of Gaussian unitary ensemble together with $vec{t} to vec{0}$ as $nto infty$ at appropriate related rates. We obtain the asymptotics of the partition function, which is described explicitly in terms of an integral involving a smooth solution to a new coupled Painlev'e system generalizing the Painlev'e XXXIV equation. The large n limit of the correlation kernel is also derived, which leads to a new universal class built out of the Psi-function associated with the coupled Painlev'e system. Joint work with Dan Dai ans Shuai-Xia Xu.