Risk-estimation based predictive densities for heteroskedastic hierarchical models

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Abstract: We consider the problem of estimating the predictive density in a heteroskedastic Gaussian model under general divergence loss. Based on a conjugate hierarchical set-up, we consider generic classes of shrinkage predictive densities with both location and scale hyper-parameters. For any $alpha$-divergence loss, we propose a risk-estimation based methodology for tuning these shrinkage hyper-parameters. Our proposed predictive density estimators enjoy optimal asymptotic risk properties that are in concordance with the optimal shrinkage calibration point estimation results established by Xie, Kou, Brown (2012) for heteroskedastic hierarchical models. These $alpha$-divergence risk optimality properties of our proposed predictors are not shared by empirical Bayes predictive density estimators that are calibrated by traditional methods such as by maximizing the likelihood or by using method of moments. We conduct several numerical studies to compare the non-asymptotic performance of our proposed predictive density estimators with other competing methods and obtain encouraging results.