

# Testing the Linear Mean and Constant Variance Conditions in Sufficient Dimension Reduction

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**Abstract:** Sufficient dimension reduction (SDR, for short) methods characterise the relationship between the response and the covariates, through a few linear combinations of the covariates. Extensive techniques are developed, among which the inverse regression based methods are perhaps the most appealing in practice because they do not involve multi-dimensional smoothing and are easy to implement. However, these inverse regression based methods require two distributional assumptions on the covariates. In particular, the first-order methods, such as the sliced inverse regression, require the linear conditional mean (LCM) assumption, while the second-order methods, such as the sliced average variance estimation, additionally require the constant conditional variance (CCV) assumption. We propose to check the validity of the LCM and the CCV conditions through mean independence tests, which are facilitated by the martingale difference divergence (MDD). We suggest a consistent bootstrap procedure to decide the critical value of the test. Monte Carlo simulations as well as an application to the horse mussels dataset are conducted to demonstrate the finite sample performances of our proposal.