Ball Covariance

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Abstract: Technological advances in science and engineering have led to the routine collection of large and complex data objects, where the dependence structure among those objects is often of great interest. Those complex objects (e.g., different brain subcortical structures) often reside in some Banach spaces, and hence their relationship cannot be well characterized by most of the existing measures of dependence such as correlation coefficients developed in Hilbert spaces. To overcome the limitations of the existing measures, we propose Ball Covariance as a generic measure of dependence between two random objects in two possibly different Banach spaces. Our Ball Covariance possesses the following attractive properties: (i) It is nonparametric and model-free, which make the proposed measure robust to model mis-specification; (ii) It is nonnegative and equal to zero if and only if two random objects in two separable Banach spaces are independent; (iii) Empirical Ball Covariance is easy to compute and can be used as a test statistic of independence. We present both theoretical and numerical results to reveal the potential power of the Ball Covariance in detecting dependence. Also importantly, we analyze two real datasets to demonstrate the usefulness of Ball Covariance in the complex dependence detection.