High-dimensional Tensor Regression Analysis

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Abstract: The past decade has seen a large body of work on high-dimensional tenors or multiway arrays that arise in numerous applications. In many of these settings, the tensor of interest is high-dimensional in that the ambient dimension is substantially larger than the sample size. Oftentimes, however, the tensor comes with natural low-rank or sparsity structure. How to exploit such structure of tensors poses new statistical and computational challenges.

In this talk, we develop a novel procedure for low-rank tensor regression, namely Importance Sketching Low-rank Estimation for Tensors (ISLET) to address these challenges. The central idea behind ISLET is what we call importance sketching, carefully designed sketches based on both the responses and the structures of the parameter of interest. We show that our estimating method is sharply minimax optimal in terms of the mean-squared error under low-rank Tucker assumptions. In addition, if a tensor is low-rank with group sparsity, our procedure also achieves minimax optimality. Further, we show through numerical study that ISLET achieves comparable mean-squared error performance to existing state-of-the-art methods whilst having substantial storage and run-time advantages. In particular, our procedure performs reliable tensor estimation with tensors of dimension p = O(10^8) and is 1 or 2 orders of magnitude faster than baseline methods.