

# A pairwise Hotelling method for testing high-dimensional mean vectors

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**Abstract:** For high-dimensional small sample size data, Hotelling's T<sub>2</sub> test is not applicable for testing mean vectors due to the singularity problem in the sample covariance matrix. To overcome the problem, there are three main approaches in the literature: replacing the covariance matrix by an identity matrix, refraining from estimating the correlations so that the covariance matrix estimate is a diagonal matrix, and applying regularization methods to obtain an invertible estimate of the covariance matrix. We note, however, that both approaches may have serious limitations and only work well in certain situations. In this paper, we propose a pairwise Hotelling method for testing high-dimensional mean vectors, which, in essence, provides a good compromise between the existing two approaches. To effectively utilize the correlation information, we construct the new test statistics as a summation of Hotelling's test statistics for the covariate pairs with strong correlations and the squared t statistics for the individual covariates with little correlation with others. We further derive the asymptotic null distributions and power functions for the proposed Hotelling tests under certain regularity conditions. Numerical results show that our new tests are able to control the type I error rates, as well as to achieve a higher statistical power compared to existing methods, especially when the covariates are highly correlated. Two real data examples are also analyzed and they both demonstrate the efficacy of our pairwise Hotelling tests.