Predictive Functional Linear Models with Semiparametric Single-Index Interactions

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Abstract: When building a predictive model using both functional and multivariate predictors, it is often crucial to include the interaction between the two sets of predictors. To overcome the curse of dimensionality, we assume the interaction depends on a nonparametric, single-index structure of the multivariate predictor and reduce the dimensionality of the functional predictor using functional principal component analysis (FPCA).

We fit the model using an iterative procedure by minimizing a local quasi-likelihood using truncated FPCA series. By treating the number of FPCA scores as a tuning parameter and allowing it to diverge to infinity, we show that for a wide range of this truncation number and different bandwidths {used by the nonparametric component in the single-index interaction}, the parametric component of the model is root-n consistent and asymptotically normal. In addition, the overall prediction error is dominated by the estimation of the nonparametric function in the single-index interaction: an outcome that leads to a CV-based procedure to select the tuning parameters. We also show that the prediction error in the functional effect enjoys the {minimax} optimal rate in Cai and Hall(2006). In a crop yield prediction application, we show that our single-index interaction model yields lower prediction error than the conventional functional linear model and other competing nonlinear functional regression models.