

Manifold Fitting: An Invitation to Statistics

Abstract

This manifold fitting problem can go back to H. Whitney's work in the early 1930s (Whitney (1992)), and finally has been answered in recent years by C. Fefferman's works (Fefferman, 2006, 2005). The solution to the Whitney extension problem leads to new insights for data interpolation and inspires the formulation of the Geometric Whitney Problems (Fefferman et al. (2020, 2021a)): Assume that we are given a set $\mathcal{Y} \subset \mathbb{R}^D$. When can we construct a smooth d -dimensional submanifold $\widehat{\mathcal{M}} \subset \mathbb{R}^D$ to approximate \mathcal{Y} , and how well can $\widehat{\mathcal{M}}$ estimate \mathcal{Y} in terms of distance and smoothness? To address these problems, various mathematical approaches have been proposed (see Fefferman et al. (2016, 2018, 2021b)). However, many of these methods rely on restrictive assumptions, making extending them to efficient and workable algorithms challenging. As the manifold hypothesis (non-Euclidean structure exploration) continues to be a foundational element in statistics, the manifold fitting Problem, merits further exploration and discussion within the modern statistical community. The talk will be partially based on a recent work Yao and Xia (2019) along with some on-going progress. Relevant reference: <https://arxiv.org/abs/1909.10228>